edexcel

# Mark Scheme (Results)

## Summer 2015

## Pearson Edexcel GCE
## in Statistics 3 (6691/01)

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Summer 2015
Publications Code UA042717
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**General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL GCE MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given
   - $\lfloor$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

# June 2015
# 6691 S3
# Mark Scheme

| Question Number | Scheme | Marks |
|---|---|---|
| **1. (a)** | Label all the books from 1 – 160 (o.e.)<br>Use random numbers to select the 10 books | B1<br>B1<br>(2) |
| **(b)** | <table><tr><td>Book</td><td>*A*</td><td>*B*</td><td>*C*</td><td>*D*</td><td>*E*</td><td>*F*</td><td>*G*</td><td>*H*</td><td>*I*</td><td>*J*</td></tr><tr><td>Borrow rank</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>Page rank</td><td>1</td><td>6</td><td>4</td><td>2</td><td>8</td><td>3</td><td>10</td><td>7</td><td>5</td><td>9</td></tr><tr><td>$d^2$</td><td>0</td><td>16</td><td>1</td><td>4</td><td>9</td><td>9</td><td>9</td><td>1</td><td>16</td><td>1</td></tr></table><br>M1<br>M1<br>$r_s = 1 - \frac{6\times 66}{10(100-1)}, \left[= 1 - 0.4\right] = 0.6$ **0.6** M1,A1<br>(4) |
| **(c)** | $H_0$: $\rho = 0$ $H_1$: $\rho > 0$<br>Critical value is 0.5636<br>0.6 > cv so significant result and sufficient evidence to reject $H_0$<br>There is support for the librarian's belief<br>or there is evidence of a correlation between the number of pages in a book and the number of times it is borrowed. | B1<br>B1<br>B1ft<br>(3)<br>**Total 9** |

**Notes**

**(a)**

1st B1 for labelling\numbering\listing\using sampling frame of all 160 books

2nd B1 for use of random numbers\selection and mentioning the number 10

**(b)**

1st M1 for an attempt to rank the number of pages ( at least 4 correct) Allow reverse ranks

2nd M1 for attempt at $d^2$ row (may be implied by sight of $\sum d^2 = 66$ or 264 for reverse ranks)

3rd M1 for use of the correct formula, follow through their $\sum d^2$ if clearly stated

If answer is not correct, a correct expression is required.

A1 for 0.6 (or $-0.6$ for reverse ranks)

**(c)**

1st B1 for both hypotheses in terms of $\rho$, one tail $H_1$ (compatible with ranks) Allow use of $\rho_s$

Hypotheses just in words e.g. "no correlation" score B0.

2nd B1 for cv of 0.5636 [ If they have a two tail $H_1$ then allow 0.6485]

Allow $\pm$ for reverse ranking but must be same sign as $r_s$

If hypotheses are the wrong way around this must be B0 but 3rd B1 is possible.

3rd B1ft for a correct contextualised comment. Must mention "librarian" (or he)

or "number of pages" and "borrowing"

Follow through their $r_s$ and their cv (provided it is |cv| <1)

Don't insist on the word "positive" or "negative" for a one-tailed test

Use of "association" is B0

Independent of 1st B1 so if $|r_s|$ > |cv| must say there is sufficient evidence of …….(o.e.)

and if $|r_s|$ < |cv| must say insufficient evidence of … (o.e.) regardless of their hypotheses

| Question Number | Scheme | Marks |
|---|---|---|
| **2. (a)** | $H_0 : \mu_g - \mu_s = 1.5$ [ $g$ = in a group, $s$ = on their own ] | B1 |
| | $H_1 : \mu_g - \mu_s > 1.5$ | B1 |
| | s.e. = $\sqrt{\frac{2.1^2}{80} + \frac{1.4^2}{65}} = \left[\sqrt{0.08527...}\right] = [0.292]$ | M1 |
| | $z = \frac{8.7 - 6.6 - 1.5}{"\sqrt{\frac{2.1^2}{80} + \frac{1.4^2}{65}}"}$ | dM1 |
| | = 2.0546…. awrt 2.05(5) | A1 |
| | cv 1% one tailed = 2.3263 | B1 |
| | Not significant, accept $H_0$ | dM1 |
| | Insufficient evidence that using plan as part of a group leads to weight loss of more than 1.5 kg than using plan on one's own<br>or researcher's belief not supported | A1ft |
| | | (8) |
| **(b)** | Since sample is large Central Limit Theorem (CLT) applies | B1 |
| | No need to assume normal distribution | dB1 |
| | | (2) |
| | | **Total 10** |
| | **Notes** | |
| **(a)** | 1st & 2nd B1 for hypotheses. Accept $\mu_1, \mu_2$ or $\mu_A, \mu_B$ etc if there is some indication of which is which e.g. $G \sim N(\mu_g, 8.7)$ | |
| | 1st M1 for an attempt at se with 3 out of 4 values correct. Condone switching 2.1 and 1.4 $\sqrt{\frac{2.1^2 \text{ or } 1.4^2}{80} + \frac{1.4^2 \text{ or } 2.1^2}{65}}$ | |
| | 2nd dM1 dependent on 1st M1 for a correct numerator(must have $-1.5$) and ft their se. | |
| | 1st A1 for awrt 2.05 | |
| | 3rd B1 for $\pm$ 2.3263 or better seen or probability of awrt 0.02 | |
| | 3rd dM1 dep. on 1st M1 for a correct statement based on their normal cv and their test statistic | |
| | 2nd A1ft for correct comment in context. Must mention "plan" and "group or individual" and "1.5" or "researcher" and "belief or claim" | |
| | NB Use of cv for difference in means $D$ will have $D$ = 1.5 + 2.3263 × s.e. = awrt 2.18 and requires sight of $d$ = 2.1 with a comment for the 3rd M1 | |
| **(b)** | 1st B1 for mentioning "large samples" and "CLT" | |
| | 2nd dB1 dependent on 1st B1 for stating **no need to** assume normality (since CLT assures it) | |

| Question Number | Scheme | Marks |
|---|---|---|
| **3. (a)** | Label staff (from 1 – 16) and children (from 1 – 40)<br>Use random numbers to select<br>4 staff and 10 children | B1<br>B1<br>B1<br>(3) |
| **(b)** | $\bar{x} = \hat{\mu} =$ 31.2142… **awrt** **<u>31.2</u>**<br>$s^2 = \dfrac{26983 - 14 \times "31.2..."^2}{13}$<br>= 1026.33… **awrt** **<u>1030</u>** | B1<br>M1<br>A1ft<br>A1<br>(4) |
| **(c)** | $\dfrac{"\sqrt{1026.33...}"}{\sqrt{14}}$ , = 8.562.. **awrt** **<u>8.56</u>** | M1, A1<br>(2) |
| **(d)** | The variation within each stratum is quite small (o.e.)<br>**The difference in the means will be quite large**, (so variations from the overall mean will be large giving a larger overall s.e.) | B1<br>B1<br>(2)<br>**Total 11** |
| | **Notes** | |
| **(a)** | 1st B1 for labelling\numbering\listing staff <u>and</u> children<br>2nd B1 for use of random numbers or "randomly select" in <u>each group</u> (may be implied)<br>3rd B1 for selecting the correct number of staff <u>and</u> children<br>e.g. randomly select 4 staff and 10 children scores 2nd and 3rd B marks since randomly selecting and the "each group" is implied, | |
| **(b)** | B1 for awrt 31.2<br>M1 for a correct expression ft their $\bar{x}$ and allow transcription error in $\sum x^2$ e.g. 29683<br>1st A1ft for a fully correct expression ft their $\bar{x}$ only<br>2nd A1 for awrt 1030 | |
| **(c)** | M1 for attempting $\dfrac{"\text{their } s"}{\sqrt{14}}$ (must have 14)<br>A1 for awrt 8.56 | |
| **(d)** | 1st B1 for a suitable comment about **variation (se)** suggesting that **variation (se) within strata is less than that overall**<br>2nd B1 for a suitable **reason** about **means**, pointing out that the individuals' weights will vary a lot from the overall mean and so overall s.e. will be higher. | |

| Question Number | Scheme | Marks |
|---|---|---|
| **4. (a)** | $H_0 : \mu = 0.5 \quad H_1 : \mu \neq 0.5$ | B1 |
| | (Significance level = )10% | dB1 |
| | (0.5 is in the interval so not significant, accept $H_0$, can accept) $\mu = 0.5$ | B1 |
| | | (3) |
| **(b)** | $1.6449 \times \frac{\sigma}{\sqrt{100}} = 0.0247$ | M1 |
| | | B1 |
| | $\sigma = 0.15016$ or $\frac{10 \times 0.0247}{1.6449}$ (awrt 0.15) | A1 |
| | $0.479 \pm 1.96 \times \frac{"\sigma"}{\sqrt{150}}$ | M1 |
| | | B1 |
| | awrt **(0.455, 0.503)** | A1 |
| | | (6) |
| | | **Total 9** |
| | **Notes** | |
| **(a)** | 1st B1 for both hypotheses in terms of $\mu$. | |
| | 2nd dB1 for 10% but accept 5% if they have a one-tail test as $H_1$ | |
| | 3rd B1 for a correct comment leading to accepting $H_0$ | |
| | Ignore any 'further calculations'. | |
| **(b)** | 1st M1 for $z\frac{\sigma}{\sqrt{100}} = k$, using $n = 100$ and where $\lvert z \rvert > 1.5$ and $0.02 < k < 0.03$ | |
| | 1st B1 for 1.6449 or better in an attempt (could be $1.6449\sigma = k$ or even $1.6449\ \sigma^2 = k$) | |
| | 1st A1 for a correct expression for $\sigma$ e.g. awrt 0.15 | |
| | 2nd M1 for $\bar{x} \pm z \times \frac{\sigma}{\sqrt{150}}$ for any $z$ ( > 1) and ft their $\sigma$ and allow $\bar{x} \in (0.4633, 0.5127)$ | |
| | Allow use of letter $\sigma$ without a value. | |
| | 2nd B1 for 1.96 or better in an attempt (could be $1.96\sigma$ or even $1.96\ \sigma^2$) | |
| | 2nd A1 for awrt 0.455 and awrt 0.503 | |

| Question Number | Scheme | Marks |
|---|---|---|
| **5 (i)** | Let $R = B_1 + B_2 + B_3 + B_4 + B_5 - 5H$ so E($R$) = $-25$ (o.e.) | B1 |
| | Var($R$) = $5\times 6^2 + 5^2 \times 4^2$ $\quad R$~N($-25$, $\sqrt{580}^2$) | M1A1 |
| | P($R > 0$) = P($Z > \frac{0--25}{\sqrt{580}}$) = P($Z > 1.04$), = 0.149619…(calc) <u>or</u> 0.1492 (tables) | dM1 A1 (5) |
| **(ii)(a)** | $\bar{X} \sim \mathrm{N}\left(\mu, \frac{\sigma^2}{5}\right)$ | B1 |
| | Var($D$) = $\sigma^2 + "\frac{\sigma^2}{5}"\left[= \frac{6\sigma^2}{5}\right]$, so $\quad D \sim \mathrm{N}\left(0, \frac{6\sigma^2}{5}\right)$ | M1, A1 (3) |
| **(b)** | P($Y_1 > \bar{X} + \sigma$) = P($D > \sigma$) = $\mathrm{P}\left(Z > \frac{\sigma}{\sqrt{\frac{6}{5}}\sigma}\right)$ | M1 |
| | = P($Z > 0.912\ldots$) = 0.181(3 dp) **(*)** | A1cso (2) |
| **(c)** | Since $U_1$ and $\bar{U}$ are not independent (so variance formula cannot be used) | B1 |
| | Can be implied e.g. $U_1$ used to calculate $\bar{U}$, $U_1$ and $\bar{U}$ from same sample o.e. | (1) |
| **(d)** | Let $F = U_1 - \bar{U} = U_1 - \frac{(U_1+U_2+U_3+U_4+U_5)}{5}, = \frac{4U_1 - (U_2+U_3+U_4+U_5)}{5}$ | M1, A1 |
| | Var($F$) = $\frac{4^2\sigma^2 + 4\sigma^2}{5^2} = 0.8\sigma^2$ ,so $\quad F \sim \mathrm{N}(0, 0.8\sigma^2)$ | dM1, A1 |
| | P($F > \sigma$) = $\mathrm{P}\left(Z > \frac{\sigma}{\sigma\sqrt{0.8}}\right)$ = P($Z > 1.118..$) | M1 |
| | = 0.1314 (tables) or 0.131776…(calc) **awrt 0.131~0.132** | A1cso (6) |
| | | **Total 17** |

| | **Notes** |
|---|---|
| **(i)** | 1st B1 for E($R$) = – 25 (or 25 if their $R$ is defined the other way around)<br>1st M1 for an attempt at Var($R$) = 5Var($B$) + 25Var($H$). Condone swapping of $6^2$ and $4^2$<br>1st A1 for normal and correct variance (ft their mean )<br>2nd dM1 for attempting the correct probability and standardising with their mean and sd.<br>This mark is dependent on 1st M1 so if $R$ is not being used or M0 for variance score M0<br>If their method is not crystal clear then they must be attempting P($Z$ > +ve value) o.e<br>2nd A1 for answer in the range [0.149, 0.150] |
| **(ii)(a)** | B1 for correct distribution of $\bar{X}$ (may be implied for a correct answer for $D$)<br>M1 for correct attempt at Var($D$) (ft their Var($\bar{X}$)) [A1 needs must be fully correct] |
| **(ii)(b)** | M1 for expressing probability in terms of $D$ and standardising<br>A1cso for seeing P($Z$> 0.912..) or prob of 1 – 0.8186 (tables) or 0.180655…(calc) |
| **(c)** | B1 correct statement that should mention $U_1$ and $\bar{U}$ |
| **(d)** | 1st M1 for forming an expression in terms of $U_1 ... U_5$ only<br>1st A1 for collecting $U_1$ terms and getting in a form where Var($aX \pm bY$) can be used.<br>2nd dM1 for a correct expression for Var(their $F$). Dependent on 1st M1.<br>2nd A1 for a correct distribution for $F$<br>3rd M1 attempting a correct prob and standardising using their Var($F$), $\sigma$ must cancel<br>3rd A1cso for awrt 0.131 or 0.132 |

| Question Number | | Scheme | Marks |
|---|---|---|---|
| **6.** | **(a)** | $H_0$ : U[0, 10] is a suitable model $H_1$ : U[0, 10] is not a suitable model | B1 |
| | | <table><tr><th>$D$</th><th>$O_i$</th><th>$E_i$</th><th>$\frac{(O_i-E_i)^2}{E_i}$</th><th>$\frac{O_i^2}{E_i}$</th></tr><tr><td>0 – 4</td><td>22</td><td>40</td><td>8.1</td><td>12.1</td></tr><tr><td>4 – 7</td><td>39</td><td>30</td><td>2.7</td><td>50.7</td></tr><tr><td>7 – 9</td><td>25</td><td>20</td><td>1.25</td><td>31.25</td></tr><tr><td>9 – 10</td><td>14</td><td>10</td><td>1.6</td><td>19.6</td></tr></table> Values of $D$ Expected Freq<br>$4^{th}$ or $5^{th}$ col<br>$\chi^2 = 13.65$ | B1<br>M1A1<br>M1<br>A1 |
| | | $\nu = 3, \quad \chi_3^2(1\%) = 11.345$ | B1, B1 |
| | | [Reject $H_0$ ,] the uniform distribution over [0, 10] is not a suitable model | A1 |
| | | | (9) |
| | **(b)** | Area $\propto \pi R^2$ so $r =$ 81, – 49 = **<u>32</u>**<br>$s$ = 100 – "32" – 49 <u>or</u> 100 – 81 = **<u>19</u>** | M1, A1<br>B1ft |
| | | | (3) |
| | **(c)** | Not significant, Henry's model is suitable | M1, A1 |
| | | | (2) |
| | **(d)** | $H_0$ : The colour\region chosen for the points is independent of gender(or no assoc') | B1 |
| | | $H_1$ : The colour\region chosen for the points is dependent on gender(or assoc') | |
| | | | (1) |
| | **(e)** | $\frac{39 \times 65}{100}$ | B1 |
| | | | (1) |
| | **(f)** | Expected frequency for Yellow and Boys is 4.9 < 5 so col. must be pooled/combined.<br>[This gives a $2 \times 3$ table so $\nu = (2-1) \times (3-1) = 2$ ] | B1 |
| | | | (1) |
| | **(g)** | cv = 4.605<br>[Not significant] so the data do <u>not</u> support Phoebe's belief oe | B1<br>B1 |
| | | | (2) |
| | | | **Total 19** |
| | | **Notes** | |
| | **(a)** | 2nd B1 for the correct values for $D$ (can be implied by 40, 30, 20, and 10.)<br>1st M1 for at least 2 expected frequencies or clear use of a correct formula e.g. 0.4$N$<br>1st A1 for all the correct $E_i$<br>2nd M1 for at least 2 correct calculations from $4^{th}$ or $5^{th}$ column<br>2nd A1 for a test statistic of 13.65 (accept 13.7 to 3 sf)<br>Awrt 13.7 only scores 2nd B1M1A1M1A1<br>3rd A1 for a correct conclusion rejecting the uniform model. Award provided their test statistic > 11.345 | |
| | **(b)** | M1 for some attempt to use $\pi R^2$ to find $r$ | |
| | **(c)** | M1 for a correct statement that it is not significant<br>A1 for correctly stating that Henry's model is suitable o.e. | |
| | **(d)** | B1 Independence or association mentioned at least once if ditto marks used.<br>Allow connection but not correlation. | |
| | **(f)** | B1 for recognising there is an $Ei < 5$ **and** need for pooling/combining oe | |
| | **(g)** | 2ndB1 for correctly stating that Phoebe's belief is not supported by the data oe (depends on their cv being > 1.411) | |

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